Fermat's Last Theorem

by Chan Shih Ping

In one of his notebooks, the 17th century French mathematician Pierre de Fermat claimed that he could prove the following equation had no whole number or integer solutions:

$$x^n + y^n = z^n,\ n \geq 3,\ x, y, z > 0.$$

This famous claim, known as "Fermat's Last Theorem", has tantalized mathematicians for over 300 years, because unfortunately, Fermat was not kind enough to leave us a proof. Indeed, he made the comment that, he had discovered a "remarkable proof" of this fact, but the margin of the notebook was too narrow for him to jot it down!

As any school child knows, when $n = 2$, there are in fact infinitely many solutions. These solutions form "Pythagorean triplets", that is, numbers which represent the sides of a right-angled triangle. Some simple examples are $3^2 + 4^2 = 5^2$, $5^2 + 12^2 = 13^2$, and $8^2 + 15^2 = 17^2$. What Fermat claimed, however, was that such equations were not possible if the exponent or "power" was greater than 2. In other words, a cube cannot be written as the sum of two smaller cubes, a fourth power cannot be written as the sum of two fourth power, and so on.

Over the years, there have many attempts to prove this "Theorem", and it has been the happy hunting ground for enthusiastic mathematical amateurs. Many eminent mathematicians have also tried to prove this theorem. Some early successes were by such prominent mathematicians like Euler and Gauss. The 18th and 19th mathematicians were, initially, only able to do special cases like when $n = 3, 5, 7, 11$, or 14. Nevertheless, the list of mathematicians that contributed to these results reads like a Who's Who of 18th and 19th century mathematics. Such names like Dedekind, Kronecker, Eisenstein, and Kummer figure prominently. The last one, in particular, made great strides and showed, for the first time, how it was possible to prove the theorem for many exponents simultaneously, as opposed to the ad hoc methods that had been developed previously. He, together with Dedekind, are recognised as being the fathers of modern Algebraic Number Theory.

In the final leg of our century, Andrew Wiles of Princeton University has finally proved the theorem. His work is the culmination of a deep and difficult strand of mathematics, which started from an inspired idea by Frey in the 1980s. Frey had the brainwave that another well-known number theory conjecture, the Shimura-Taniyama-Weil Conjecture, implied Fermat's Last Theorem. He could not quite prove this, but Ken Ribet, a mathematician from Berkeley, succeeded in showing that Frey's idea was correct. Thus if one proved the Shimura-Taniyama-Weil Conjecture then Fermat's Last Theorem would follow. This is essentially what Wiles did. The Shimura-Taniyama-Weil Conjecture turns out to be stronger than is necessary to prove Fermat's Last Theorem. Wiles proved a special case of the Shimura-Taniyama-Weil Conjecture that is enough to imply Fermat's Last Theorem. The Shimura-Taniyama-Weil Conjecture had long been considered to be out of reach of current mathematical techniques so even his partial result was a tremendous success.

His method of proof is so complicated and difficult that his original proof, announced in 1993 and making headlines in several leading newspapers, was found to be incomplete. It is reassuring to note that even the greatest scientists are human after all and can make mistakes - a small comfort to us mere mortals. However, together with his student Richard Taylor from the University of Cambridge, he was able, in 1994, to patch the gap in his proof. The amendment to the proof has been warmly received, because based on further work of another Wiles' student Fred Diamond, it now seems possible to prove an even more lucrative target, namely, the full Shimura-Taniyama-Weil Conjecture. Wiles' manuscript has been making the rounds of the leading mathematicians of the world, and the general consensus seems to be that it is correct.

Fermat's Last Theorem may not seem to be a deeply earth-shattering result. Its importance lies in the fact that

Pierre de Fermat 1608 - 1665

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it has captured the imagination of some of the most brilliant minds over the last 300 years and their attempts at solving this conundrum, no matter how incomplete or futile, has led to the development of some of the most important branches of modern mathematics.

Several conferences will probably take place in the near future, where Wiles' proof will be expounded to the general mathematical public. No doubt, the proof will be held up to close scrutiny, and it will be gone over with a fine tooth-comb. This should dispel all doubt to the accuracy of Wiles' proof, and the ghost of Fermat will finally be laid to rest. M²